Sr.No. 14809

Exam.Code: 211004
Subject Code : 4637

M.Sc.(Mathematics) - 4th Semester

(2720)

Paper - MATH-576: Integral Transforms

Time Allowed: 2 hrs. **Max. Marks: 100**

Note: Attempt any four questions. All questions are of equal marks.

1.(a) Find the finite Fourier cosine transform of $f(x) = \frac{\pi}{3} - x + \frac{x^2}{2\pi}$. $(8\frac{3}{4})$

(b) Find the finite Fourier sine transform of $f(x) = x^2,\ 0 < x < 4$. $(8\frac{3}{4})$

(c) Find the Fourier transform of

$$f(x) = \begin{cases} 1\,, & |x| < a \\ 0\,, & |x| > a \end{cases}$$ $(7\frac{1}{2})$

2.(a) Using the Fourier sine transform, sovle the partial differential equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

With the boundary conditions $u = u_0\ when\ x = 0\,, t > 0$, and the initial condition

$u = 0\ when\ t = 0\,, x > 0$. $(12\frac{1}{2})$

(b) Solve the integral equation $\int_0^\infty F(x)\cos px\, dx = \begin{cases} 1-p, & 0 \le p \le 1 \\ 0, & p > 1 \end{cases}$.

Hence deduce that $\int_0^\infty \frac{\sin^2 t}{t^2}\, dt = \frac{\pi}{2}$. $(12\frac{1}{2})$

3.(a) Using Laplace transforms, solve the equation

$y'' - 3y' + 2y = 4t + e^{3t}\,, when\ y(0) = 1\ and\ y'(0) = -1.$ $(12\frac{1}{2})$

(b) Using Laplace transform solve the following IBVP:

$U_t = U_{xx}\,, \qquad 0 < x < 1\,, \quad t > 0$

$U(0,t) = 1,\quad U(1,t) = 1, \qquad t > 0$

$U(x,0) = 1 + \sin \pi x\,, \qquad 0 < x < 1.$ $(12\frac{1}{2})$

4.(a) Determine steady state temperature distribution of thin rectangular plate bounded by the lines $x = 0,\ x = l,\ y = 0,\ y = b$, the edges $x = 0,\ x = l,\ y = 0$ are maintained

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at zero temperature, where as the edge $y = b$ is kept at temperature $f(x)$. $(12\frac{1}{2})$

(b). A string is stretched and fastened to two points l distance apart. Motion is started by displacing the string in the form of $y = a\sin\left(\frac{\pi x}{l}\right)$ from which it is released from rest at time $t = 0$. Show that the displacement of any point at a distance x from the end at time t is given by

$$y(x,t) = a\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi ct}{l}\right).$$ $(12\frac{1}{2})$

5.(a) Find the Hankel transform of

$$\frac{d^2f}{dx^2} + \frac{1}{x}\frac{df}{dx} - \frac{n^2}{x^2}f.$$ $(8\frac{3}{4})$

(b) Find the Hankel transform of $\frac{df}{dx}$, $when\ f = \frac{e^{-ax}}{x}$ $and\ n = 1$. $(8\frac{3}{4})$

(c) State and prove Parseval's theorem for Hankel transforms. $(7\frac{1}{2})$

6.(a) Use Hankel transform to solve the differential equation

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0 \quad for\ r \geq 0, z \geq 0$$

satisfying the following conditions:

$V \to 0\ as\ z \to \infty\ and\ r \to \infty$,

$V = f(r)\ on\ z = 0, r \geq 0$. $(12\frac{1}{2})$

(b) Prove that the finite Hankel transform of $f(x),\ 0 \leq x \leq 1\ is\ s^{n-m}J_m(s)$ where

$$f(x) = \frac{2^{1+n-m}}{\Gamma(m-n)}.x^n(1-x^2)^{m-n-1}.$$ $(12\frac{1}{2})$

7(a). Find Z-transform of the followings:

(i) $\frac{1}{n(n+1)}$ (ii) $e^{4t}\cos t$ (iii) $n\cos n\theta$ $(3\frac{3}{4} + 3\frac{3}{4} + 5)$

(b) State and prove convolution theorem for Z-transforms. $(6\frac{1}{4})$

(c) Find the convolution $f * g$ where $f(n) = n(n-1)$, $g(n) = 3^n$. $(6\frac{1}{4})$

8(a). Find the inverse Z-transform of $\frac{z}{z^2+7z+10}$ $(6\frac{1}{4})$

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(b) Find the inverse Z-transform of $\frac{z^3-20z}{(z-2)^3(z-4)}$. (6¼)

(c) Use Z-transform to solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n \ given \ y_0 = y_1 = 0$. (12½

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